# Quantitative principles in biological systems

## 7. Morphogenesis and information theory

Spring 2026

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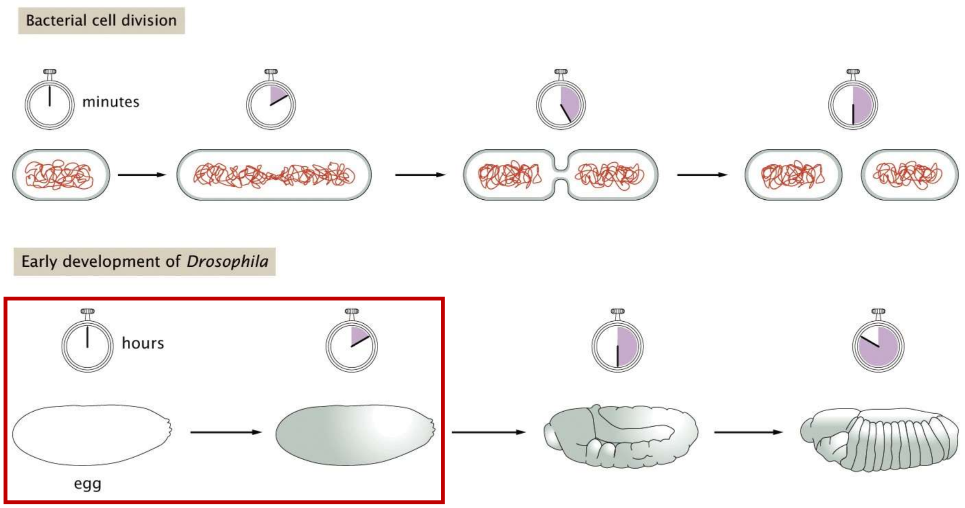
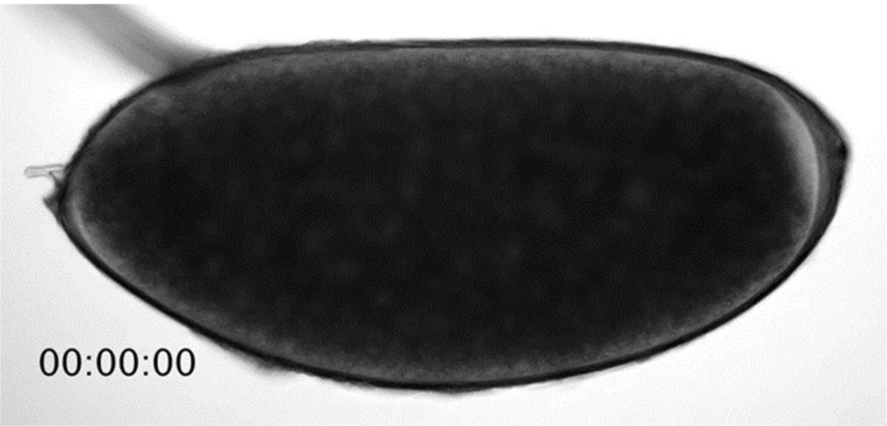
## Review



Phillips et al

Fruit fly embryogenesis will be our model system for representing information.



Thomas Gregor lab

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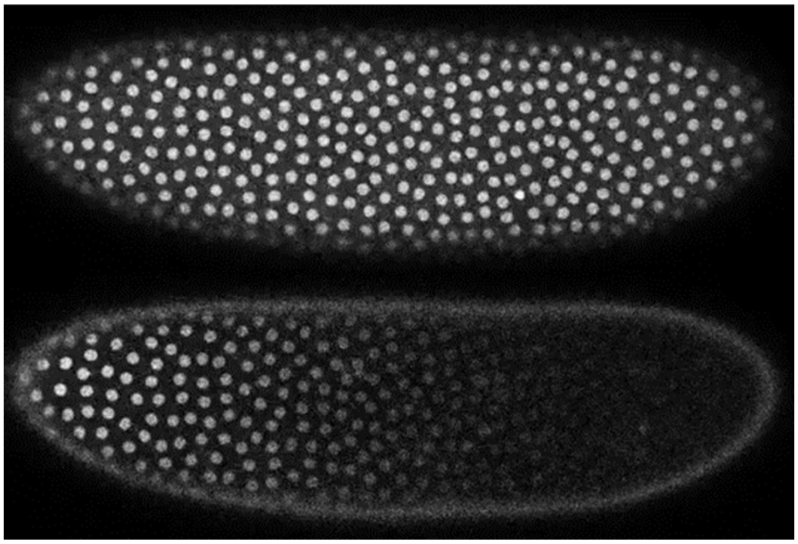

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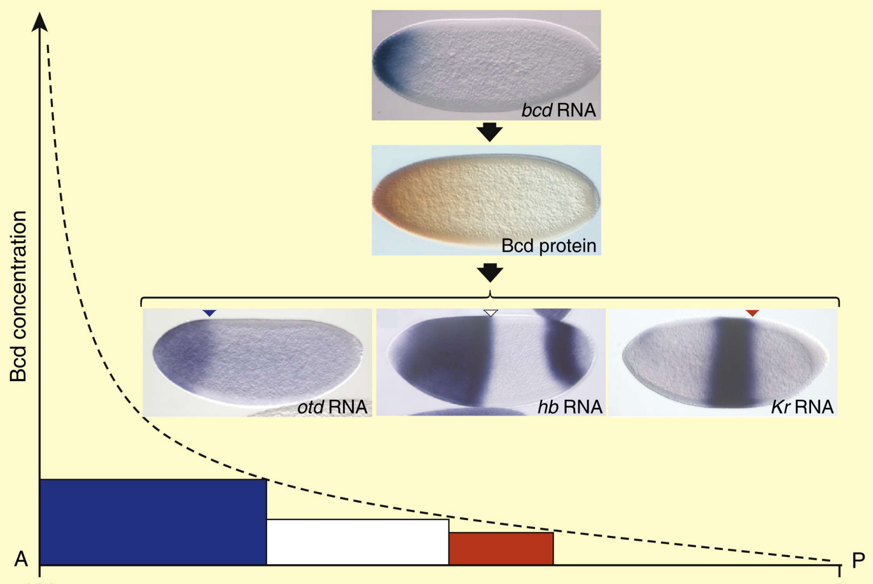
## Maternal inputs lead to morphogen gradients.



Porcher and Dostatni. *Curr Biol* (2010)

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## Reaction diffusion

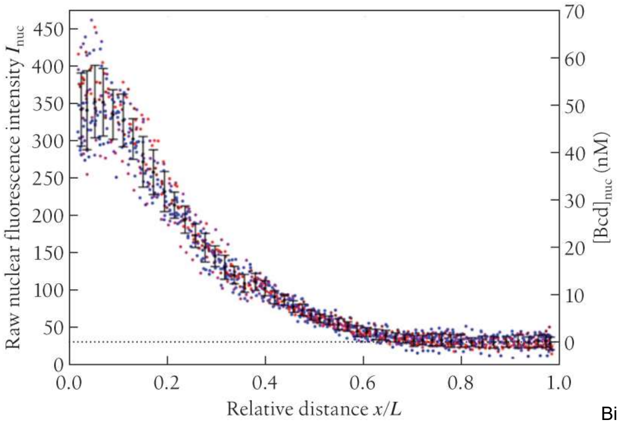
$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2} - \frac{1}{\tau}c(x,t)$$

$$-D\frac{\partial c(x,t)}{\partial x}\bigg|_{x=0} = R$$

$$\frac{\partial c(x,t)}{\partial x}\bigg|_{x=L} = 0$$

$$c(x) = \frac{R\tau}{\lambda}e^{-x/\lambda}$$

$$\lambda = \sqrt{D\tau}$$

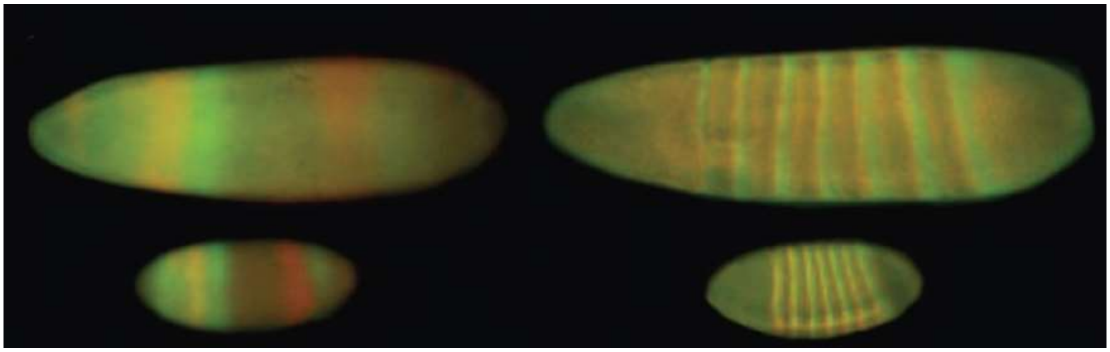


Bialek

## Reaction diffusion – No scaling? Not robust?

$$\frac{\partial c(x,t)}{\partial t} = D\frac{\partial^2 c(x,t)}{\partial x^2} - \frac{1}{\tau}c(x,t) \qquad c(x) = \frac{R\tau}{\lambda}e^{-x/\lambda}$$

$$\lambda = \sqrt{D\tau}$$



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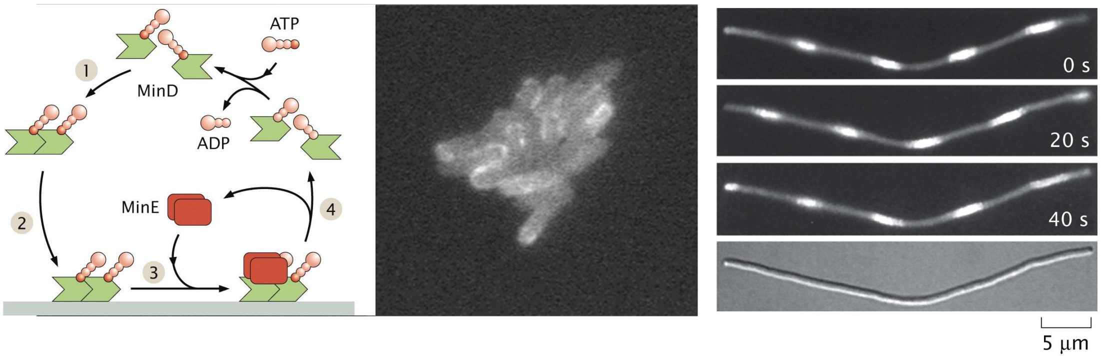
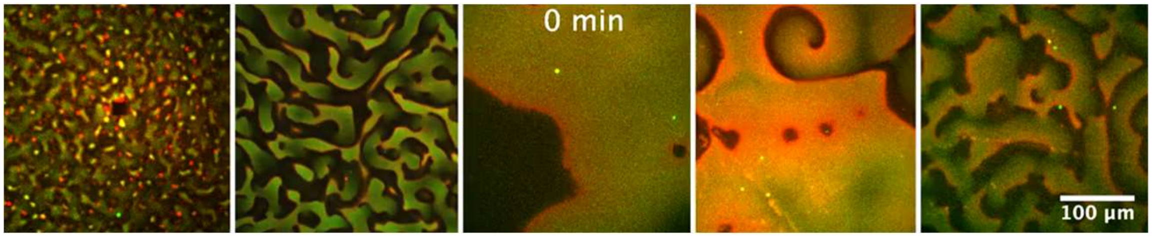
## Detour: The Min system oscillates to determine division site.



Fabai Wu et al. *Nat Nanotechnol* (2015)
Phillips et al

# Detour: Min system in vitro forms different classes of patterns.



Brauns et al. *Nat Commun* (2021)

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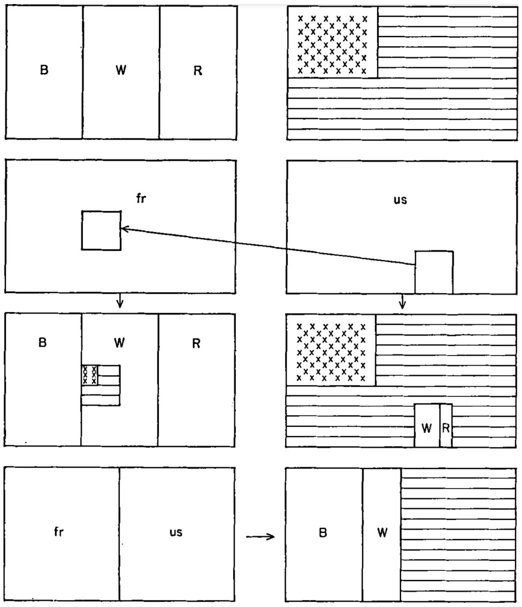
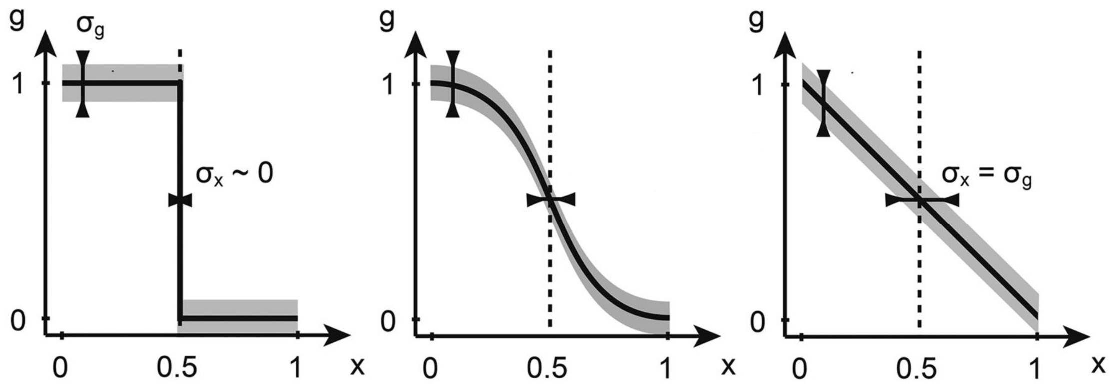
# “Positional information”



Wolpert. *J Theoret Biol* (1969)

## Positional information



Tkacik et al. *Genetics* (2015)

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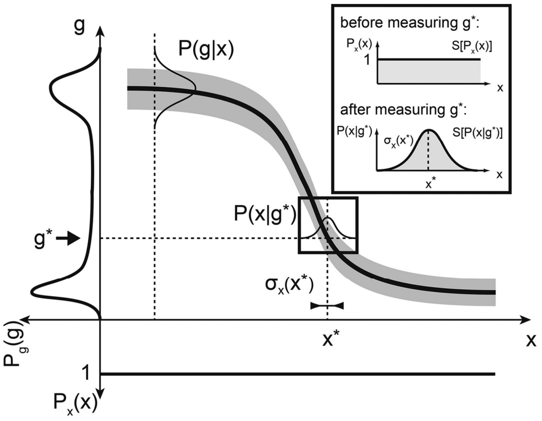
## Information theory

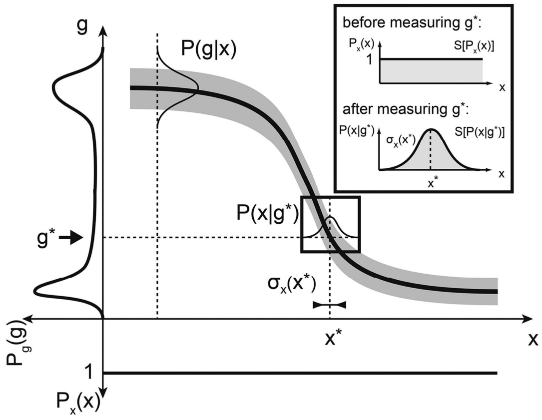


$$I(g,x) = S\big(p(x)\big) - S\big(p(x|g)\big)$$

$$S\big(p(x)\big) = -\sum_{x} p(x)\log p\,(x)$$

Tkacik et al. *Genetics* (2015)

## Information theory



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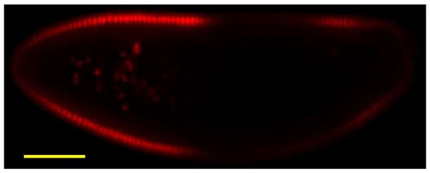
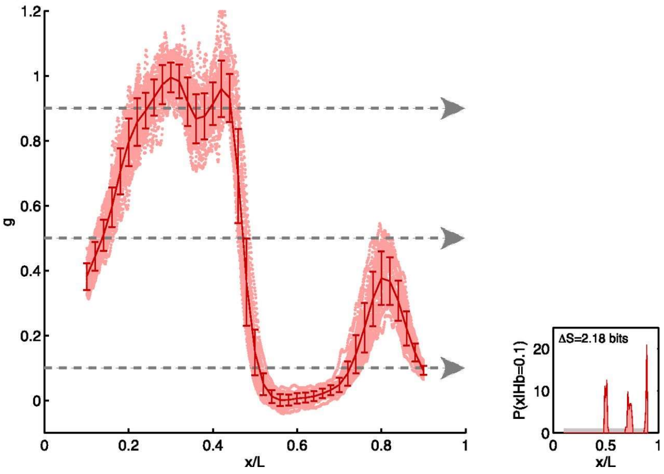
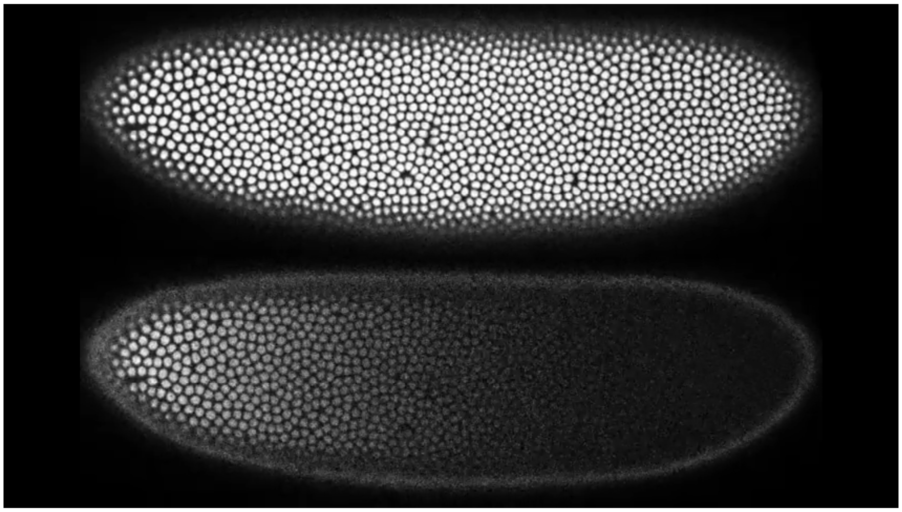




Dubuis et al. *PNAS* (2013)

## Positional error



≈60 cells → ≈6 bits to specify every cell

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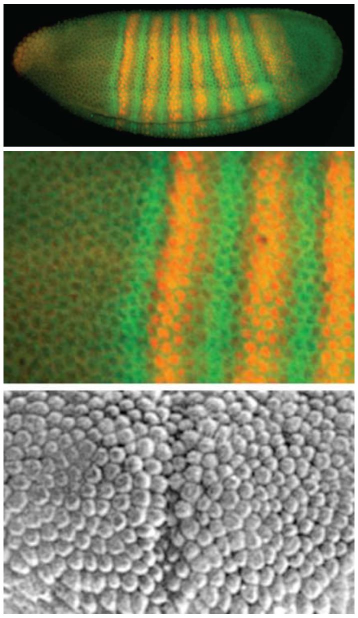
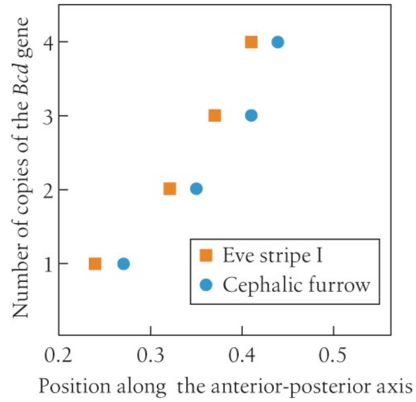
## Maternal inputs → gap genes → pair rule genes → body plan → positional error





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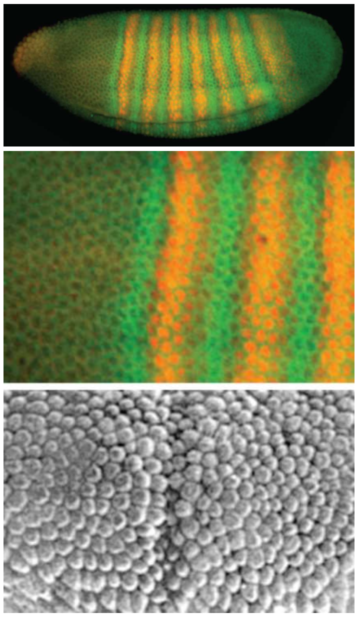
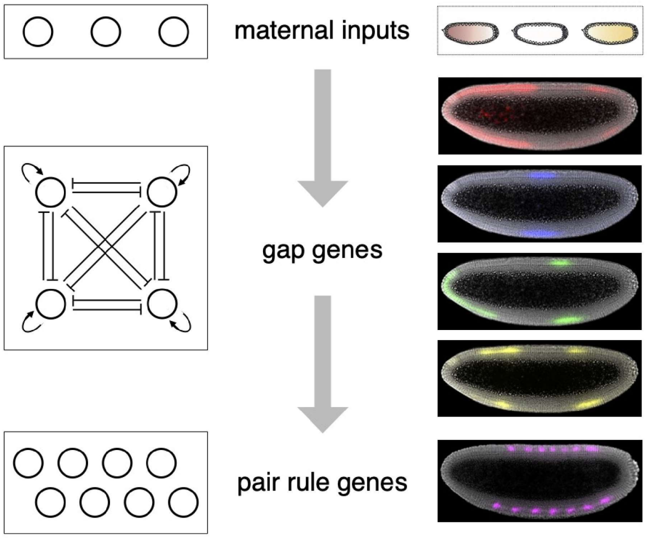
## Maternal inputs → gap genes → pair rule genes → body plan → positional error





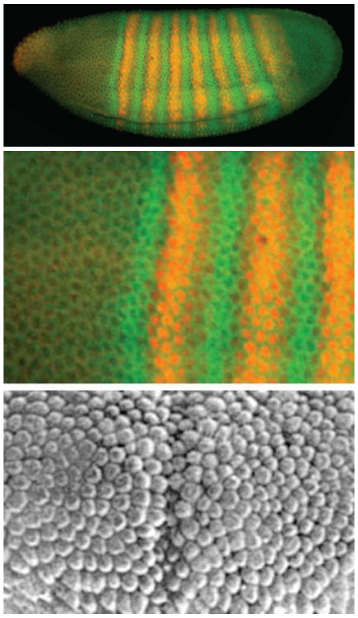
maternal inputs

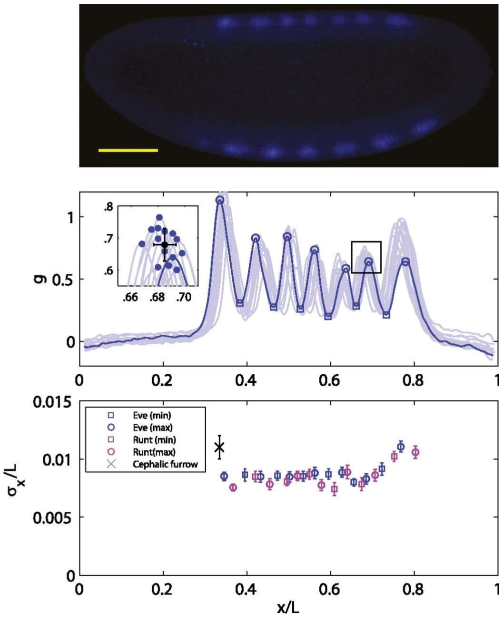
gap genes

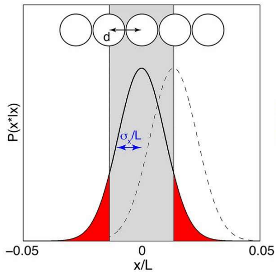
pair rule genes

Bialek

# Maternal inputs → gap genes → pair rule genes → body plan → positional error







≈ 1% positional error

Dubuis et al. *PNAS* (2013)

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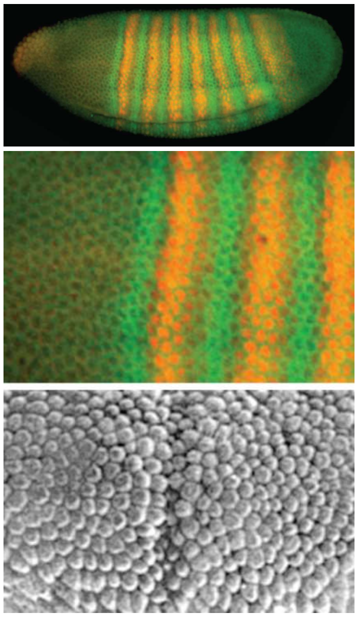
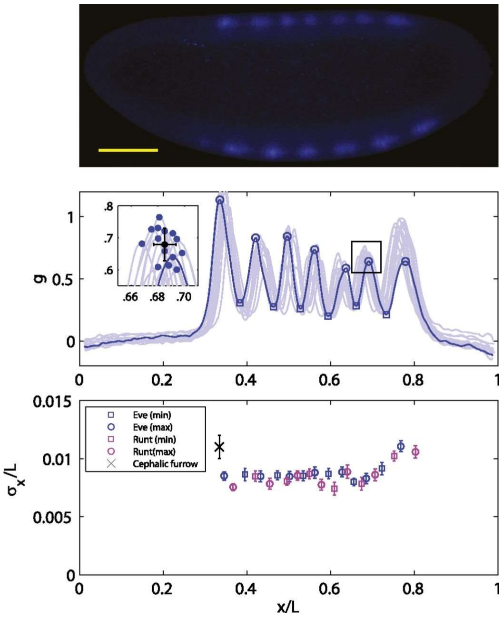
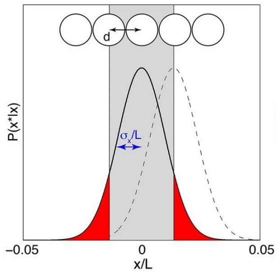
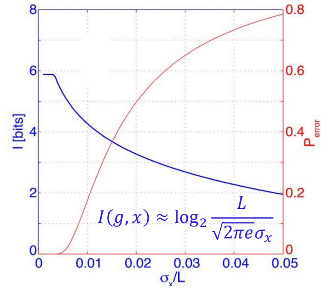


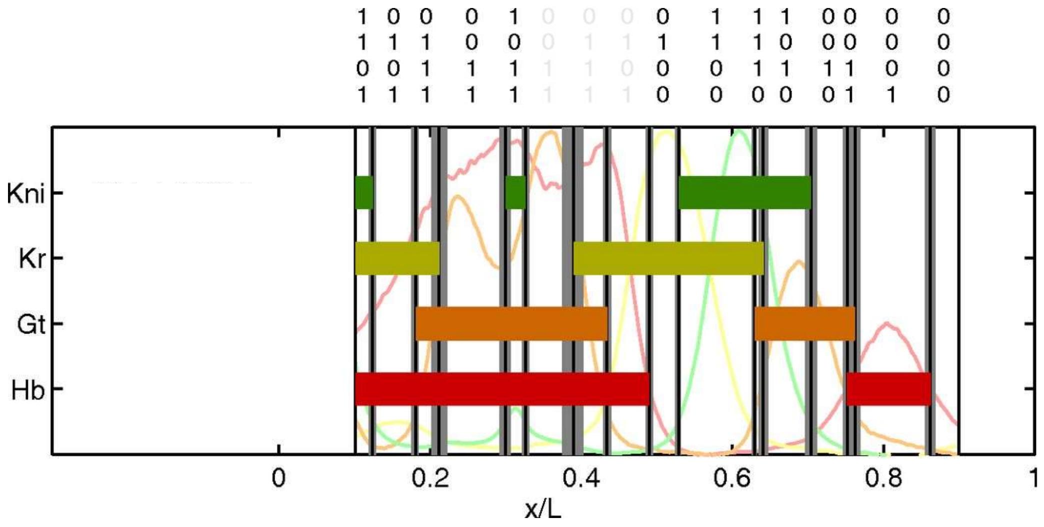






Dubuis et al. *PNAS* (2013)

## How many bits of information?



Dubuis et al. *PNAS* (2013)

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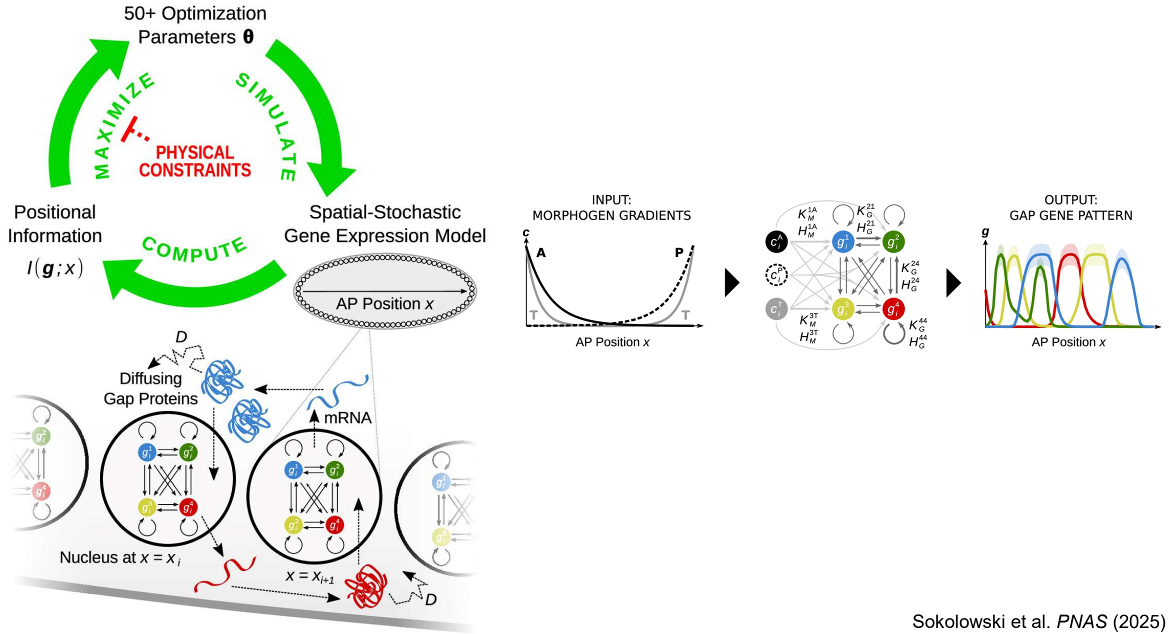
## Maximizing positional information predicts gap gene profiles without parameters.



Sokolowski et al. *PNAS* (2025)

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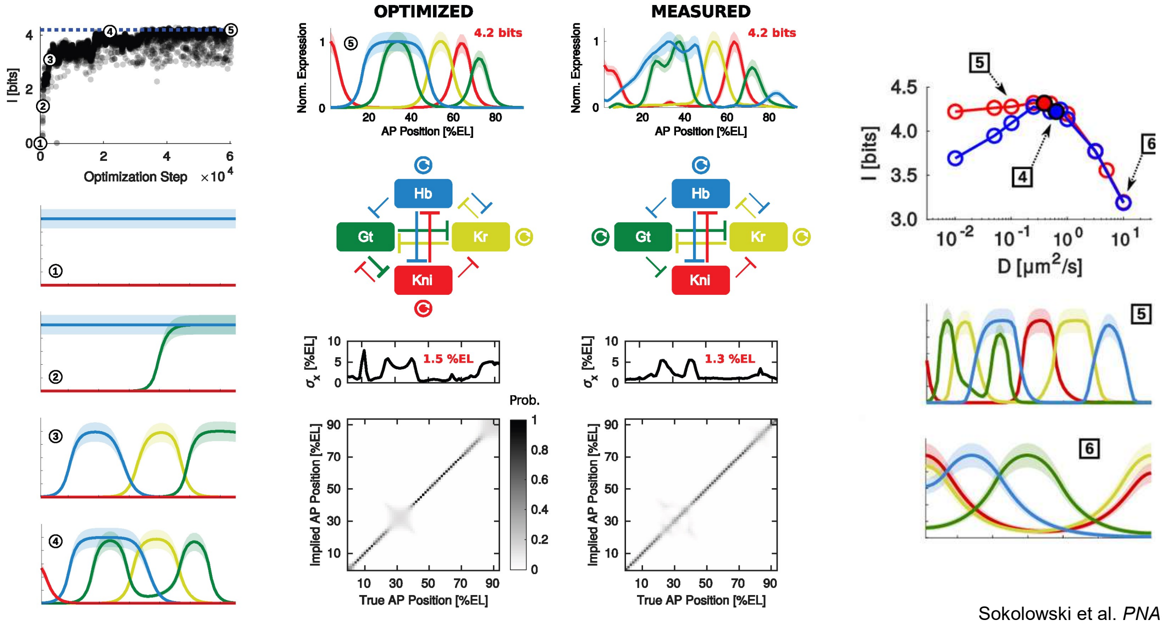

Sokolowski et al. *PNAS* (2025)

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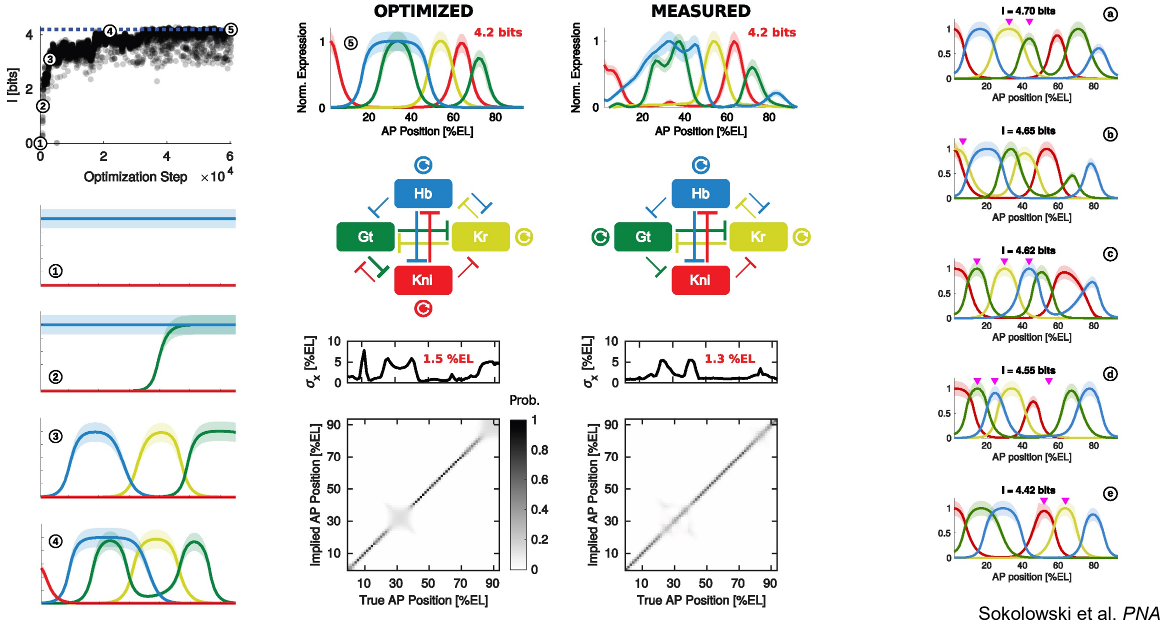

Sokolowski et al. *PNAS* (2025)