LECTURE 2. CHEMOTAXIS AND CHEMICAL REACTION NETWORKS

Chemical reaction networks

Standard conditions

Escape over a barrier

It is possible to derive the Arrhenius equation by considering a particle in a finite potential well and calculating its escape rate over the potential barrier. The problem can be posed using the Fokker-Planck formalism introduced above, and was first solved by Kramer in the 1940s. The solution gives both the exponential dependence on temperature as well as the prefactor, which depends on the shapes of the potential well and the barrier. The derivation is beyond the scope of this class, but I want to mention one of the key steps that will help us visualize the process.

Calculating the rate amounts to determining the probability distribution of the random trajectories $P[\xi(t)]$. Since the particles are in a potential well $U(x)$, trajectories that traverse high energy paths are exponentially less probable. In fact, it turns out we must evaluate an integral in the form

$$\int_{x=a}^{x=c} e^{\frac{U(x)}{kT}} dx,$$

where a denotes the position of the initial well and c the well on the other side of the barrier. The key idea is that the integral is dominated by the peak of the integrand, so that the integral can be evaluated by expanding about $x = b$, where b is the position of the barrier. Since b is a peak, the first derivative is zero at b, so the integral becomes approximately

$$e^{\frac{U(b)}{kT}} \int_{-\infty}^{\infty} e^{-\frac{|U''(b)|}{2kT}(x-b)^2} dx.$$

The integrand now is simply a Gaussian and evaluates to give a factor of $\sqrt{|U''(b)|}$. This factor describes the shape of the potential around the barrier and enters into the prefactor in the Arrhenius law.

This approximation is known as a saddle point approximation because in multiple dimensions, b will be a saddle point – it will be the maximum along the reaction coordinate but the minimum in all other orthogonal directions, as visualized in this landscape. In sum, the escape rate over the barrier is exponential in barrier height with a prefactor that depends on the shape of the saddle around the barrier.

Implications: Probability to observe the transition state is low.

Michaelis-Menten

Separation of timescales

Adaptation

Buffer variable –
slow methylation + CheR at saturation => output independent of ligand concentration

Cooperativity
Depletion of intermediate states